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Testing for cointegration using the Johansen approach:
Are we using the correct critical values?

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Abstract:

This paper presents Monte Carlo simulations for the Johansen cointegration test which indicate that the critical values applied in a number of econometrics software packages are inappropriate. This is due to a confusion in the specification of the deterministic terms included in the VECM between the cases considered by Osterwald-Lenum (1992) and Pesaran, Shin and Smith (2000). The result is a tendency to reject the null of no cointegration too often. However, a simple adjustment of the critical values is enough to deal with the problem.

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I. Introduction

The Johansen (1988) method of testing for the existence of cointegrating relationships has become standard in the econometrics literature. However, its application is made problematic by inconsistencies in the assumptions made about the Vector Error Correction model (VECM) used to construct the test statistics and the appropriate critical values for the tests applied. This paper seeks to examine the application of the Johansen method in four widely used econometrics software packages. Our conclusion is that, unless we are careful to apply the correct critical values, the Johansen method can lead to significantly misleading results. What needs to emphasised from the start is that this paper is *not* arguing the case that the critical values need to be modified to take into account the sample size or the lag length of the VECM. There may be perfectly good reasons for making such modifications. However, the central point of this paper is that it is the specification of the deterministic terms in the VECM which is the most important factor in the choice of critical values.

One of the main problems which arises in the application of the Johansen method is that the specification of the deterministic terms which enter the VECM relationship used to construct the test statistics is often left unclear. There are two classifications in common use which are summarised in Table One. The first[1] is that associated with Johansen's original work and with the Monte Carlo study of Osterwald-Lenum (1992) while the second is associated with the work of Pesaran, Shin and Smith (2000) (PSS).

[Table One here]

[1] We refer to this classification as the Osterwald-Lenum classification since this usage is common in the literature due to the important paper by Osterwald-Lenum (1992) which generated tables of critical values for these cases. However, the original discussion of these cases goes back to Johansen (1988).

Three of the five cases are identical in the two classification systems. Case 0 of Osterwald-Lenum is identical to case I of PSS since in both cases there are no deterministic terms included in the VECM. Case 1* of Osterwald-Lenum is identical to case II of PSS. Here both cases include an intercept in the VECM but this is restricted so that it is included in the levels part only i.e. the cointegrating vector. Since the intercept in differences is restricted to zero, this is appropriate when there are no trends evident in the individual series. Finally, case 2* of Osterwald-Lenum is identical to case IV of PSS. Here both cases include an unrestricted intercept in the VECM and a linear trend which is restricted so that it is included in the levels part only i.e. as part of the cointegrating vector.

The classification systems differ in two cases. Case 1 of Osterwald-Lenum and case III of PSS both allow an unrestricted intercept in the VECM. However, case 1 of Osterwald-Lenum also allows for a restricted linear trend (though it is not clear what exactly is the nature of the restrictions applied). Similarly, case 2 of Osterwald-Lenum and case V of PSS both allow for unrestricted intercepts and linear trends in the VECM but Osterwald-Lenum also allows for a restricted quadratic trend. These differences are crucial when it comes to the appropriate choice of critical values for the tests.

II. Comparison of four econometrics packages

Following the discussion in the previous section, it is useful to compare the output for cointegration tests from four popular econometrics software packages. The packages examined are Microfit version 4.0, EViews version 5.1, PcGive version 11.1 and STATA version 9. The results in Table Two are based on cointegration tests carried out on an artificial data set comprising two independent random walk variables. In each case the specification adopted was to conduct a cointegration test with an unrestricted intercept but no trend in the VECM (case 1 of Osterwald-Lenum and case III of PSS). For the sake of brevity, only the trace statistic has been presented. However, where available, the output for the maximum eigenvalue statistic produces identical patterns.

[Table Two here]

A striking feature of Table Two is that, although each software package produces identical test statistics, they differ in terms of the critical values and/or the p-values reported. The fact that the test statistics are identical presumably means that the same VECM is being used for their construction[2]. However, the critical values and/or p-values depend on how these are being matched to the specification of deterministic components given in Table One.

To determine the assumptions being made about the nature of the VECM it is necessary to go back to the source of the critical values and/or p-values reported. Microfit uses the tables of critical values reported in Pesaran, Shin and Smith (2000). In particular the 5% critical value of 17.86 for $H_0 : r = 0$ can be found in Table 6(c) p. 339 and is appropriate for case III of PSS which assumes unrestricted intercepts but no trends. The 5% critical value of 15.49 for $H_0 : r = 0$ reported by EViews can be found in the computer programme associated with MacKinnon, Haug and Michelis (1999) (MHM) and is appropriate for case 1 of Osterwald-Lenum. The p-value of 0.657 for $H_0 : r = 0$ reported in the PcGive output is derived from the response surfaces estimated by Doornik (1998). The fact that this is virtually identical to the p-value reported in the EViews output suggests strongly that this too is appropriate for case 1 of the Osterwald-Lenum classification. For STATA the test statistics are identical to those given by the other packages and the critical values are close (but not identical) to the EViews critical values. Overall, therefore it appears that the critical values given by Microfit are noticeably different from those given (or implied) by the other three packages.

[2] In fact it is possible to replicate these test statistics exactly using procedure described in Davidson and MacKinnon (2004) pp. 640-642 and assuming a VECM with an unrestricted intercept only.

The relationship between the test statistics and the critical values observed in Table Two for the case of unrestricted intercepts in the VECM is also evident when unrestricted trends are included. Table Three presents output from the four regression packages for the case where the VECM includes an unrestricted intercept and an unrestricted trend. Again, the trace test statistics are identical but the critical values differ substantially. For the Microfit output, the critical values given can be identified as deriving from Pesaran, Shin and Smith (2000) table 6(e) p.341. For EViews the critical values are identical to those for Osterwald-Lenum case 2 taken from the MHM computer programme. Again, the PcGive p-values are slightly different from the EViews p-values but sufficiently close for us to believe that they are derived from the same underlying distribution. The STATA critical values are again close (but not identical) to the EViews critical values.

Given these results, the natural question which arises is which, if any, of these software packages is using the 'correct' set of critical values. A possible method for tackling this question is to make use of Monte Carlo methods to estimate empirical critical values and to compare these with the estimates presented. The Monte Carlo design in this paper is one of the standard data generation processes (DGPs) adopted by Engle and Granger (1987). The form of the DGP is given by equations (1) and (2).

$$y_t + x_t = u_{1,t} \qquad \Delta u_{1,t} = \varepsilon_{1,t} \tag{1}$$

$$y_t + 2x_t = u_{2,t} \quad \left(1-\theta L\right)u_{2,t} = \varepsilon_{2,t} \tag{2}$$

where $t = 1,...,T$ and $0 < \theta \le 1$. $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$; $t = 1,...,T$ are independent identically distributed $N\left(0,1\right)$ variables. The variables y and x can be written as linear combinations of the u terms $y_t = 2u_{1,t} - u_{2,t}$ and $x_t = -u_{1,t} + u_{2,t}$. This formulation ensures that both y and x individually contain a unit root but there is a cointegrating relationship between y and x, with cointegrating parameter -2, providing $\theta < 1$. Thus each variable has both a unit root component and a transitory autoregressive component when $\theta < 1$.

This framework has since been used in numerous other papers. For example, this DGP forms the basic testing framework for a series of papers on the role of data span vs. number of observations in determining the power of cointegration tests (cf. Hooker (1993), Lahiri and Mamingi (1995) and Otero and Smith (2000)). Alternatively, for a more recent example, see Ericsson and MacKinnon (2002) who use this DGP explicitly to investigate the power of alternative cointegration tests.

Table Four presents empirical estimates of the critical values based on 10,000 replications carried out using EViews. The sample size is set at 1,000 throughout. The cointegration test statistics performed were for EViews case 3 and were drawn directly from the EViews output tables. For the case of no cointegration we set $\theta = 1$ and estimate the critical values using the empirical percentiles of the trace statistic for the null $H_0 : r = 0$ where r is the number of cointegrating vectors. For the case of a single cointegrating vector we set $\theta = 0.85$ and estimate the critical values using the empirical percentiles of the trace statistic for the null $H_0 : r \leq 1$. The resulting estimates are much close to those reported by PSS for their case III than to the critical values for Osterwald-Lenum's case 1. When we use MHM's more accurate estimates of PSS case III critical values then our estimates are even closer. Therefore the conclusion is that when evaluating the significance of the EViews trace statistic, it is better to use the PSS case III critical values rather than the critical values given in the EViews output which are appropriate for Osterwald-Lenum case I. By extension the same argument holds when interpreting the PcGive trace estimate of the trace statistic where it is better to use the p-values for PSS case III rather than the values given which appear to match Osterwald-Lenum case I. Although the results are not reported for the sake of brevity, exactly the same arguments apply to the maximum eigenvalue statistics.

[Table Four here]

III. Implications for rejection frequencies

The results in the previous sections have established that the appropriate critical values for Johansen tests which include unrestricted trends in the VECM correspond to case III of PSS. The best numerical estimates of these critical values and the associated p-values are given by the tables in MacKinnon et al (1999) and the associated programme which is downloadable from www.econ.queensu.ca/jae . The question remains however, as to whether the particular critical values used make an important difference for empirical research.

To address this question we proceed in two stage. First, we present Monte Carlo results for rejection frequencies of $H_0: r = 0$ against $H_1: r \geq 1$ using the MHM estimates of PSS case III and V critical values and MHM estimates of the critical values from the Osterwald-Lenum case 1 and 2 classification. These are given in Table Five which indicates that the use of the Osterwald-Lenum classification substantially increases the nominal size of the tests. For the unrestricted intercept – no linear trend case (EViews case 3) the rejection frequency is more than double the size of the test for the trace test. The rejection frequency is lower for the maximum eigenvalue test but still well above the correct size of the test. When the PSS case III critical values are used the rejection frequency is close to the correct size. If we compare the rejection frequencies for the unrestricted intercept – unrestricted trend case (EViews case 5) then the rejection frequency when the Osterwald-Lenum classification is used is even higher relative to the correct size of the test. Again when the PSS case V critical values are used the rejection frequency is close to the correct size of the test.

[Table Five here]

A potentially more serious problem emerges when we consider $H_0: r \leq 1$ against $H_1: r = 2$ in systems with a single cointegrating vector. In this case the simulation model consisted of equations (1) and (2) with $\theta = 0.85$. The results of simulations of this model are given in Table Six. These show that the rejection frequencies are well above the correct size of the test in both cases. For EViews case 3 the rejection frequencies are over 30% and for case 5 over 60% with a nominal size of the test equal to only 5%. Thus there is a real danger of detecting spurious ‘second’ cointegrating vectors when we use the incorrect critical values.

[Table Six here]

Another interpretation of these results can be made using the relationship between the Osterwald-Lenum classification and the p-values for the distribution of the PSS statistics. If the PSS distribution is correct then the percentile corresponding to the Osterwald-Lenum classification should give the rejection frequency when these are used in place of the PSS values. To examine this hypothesis we compare the percentiles of the PSS distribution with empirical rejection frequencies based on 10,000 replications for $H_0 : r = 0$ against $H_1 : r > 0$ using the trace test with systems of dimension 1, 2, 3, 4 and 5. The DGP used assumes that the variables entering the VECM are independent random walks and we calculate trace test statistics assuming (a) an unrestricted intercept in the VECM and (b) an unrestricted intercept and an unrestricted trend in the VECM. The results, given in Table Seven, and illustrated in Figure One show a close match between the predicted and actual rejection frequencies and act as further confirmation that the PSS distribution is appropriate.

[Table Seven and Figure One here]

IV. Conclusions

The specification of the deterministic terms in the VECM used to construct Johansen cointegration tests is of crucial importance. This is particularly the cases where the VECM includes unrestricted intercepts and/or trends. Case 1 of Osterwald-Lenum and case III of PSS have very different associated critical values and mismatching the critical values to the test being employed can lead to incorrect conclusions. In particular, the use of the critical values for the Osterwald-Lenum case 1 classification when the PSS case III VECM is estimated means that the probability of detecting spurious cointegrating vectors is noticeably higher than the nominal size of the test. The same result holds when we use critical values for Osterwald-Lenum case 2 and the test statistics are constructed using the PSS case V VECM.

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TABLE ONE: Deterministic Terms in the VECM

Osterwald-Lenum Case	Intercept	Linear Trend	Quadratic Trend	PSS Case	Intercept	Linear Trend	Quadratic Trend
0	N	N	N	I	N	N	N
1*	R	N	N	II	R	N	N
1	U	R	N	III	U	N	N
2*	U	R	N	IV	U	R	N
2	U	U	R	V	U	U	N

N = not included, U = unrestricted coefficient, R = restricted coefficient.

TABLE TWO: Trace test output from four econometrics software packages

Microfit output

Null	Alternative	Statistic	95% Critical Value	90% Critical Value
r = 0	r>= 1	6.3626	17.8600	15.7500
r<= 1	r = 2	1.8508	8.0700	6.5000

EViews output

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None	0.008983	6.362608	15.49471	0.6527
At most 1	0.003695	1.850791	3.841466	0.1737

PcGive Output

H0: rank<=	Trace Test	[Prob]
0	6.3626	[0.657]
1	1.8508	[0.174]

STATA Output

maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value
0	2	-1435.8	.	6.3626*	15.41
1	5	-1433.55	0.00898	1.8508	3.76
2	6	-1432.62	0.00369		

TABLE THREE: Trace test output from four econometrics software packages

Microfit output

Null	Alternative	Statistic	95% Critical Value	90% Critical Value
r = 0	r>= 1	22.3291	23.8300	21.2300
r<= 1	r = 2	4.0321	11.5400	9.7500

EViews output

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.035932	22.32908	18.39771	0.0134
At most 1 *	0.008032	4.032115	3.841466	0.0446

PcGive Output

H0: rank<=	Trace Test	[Prob]
0	22.329	[0.012]
1	4.0321	[0.045]

STATA Output

maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value
0	4	-1435.63	.	22.3291	18.17
1	7	-1426.48	0.03593	4.0322	3.74
2	8	-1424.46	0.00803		

TABLE FOUR: Estimates of 5% critical value for Johansen Trace Test in a VECM of order 2

	$H_0 : r = 0$ $H_1 : r \geq 1$	$H_0 : r \leq 1$ $H_1 : r = 2$
Unrestricted Intercepts, No Trend		
Monte Carlo Estimate	18.15	8.23
PSS Tables Case III	17.86	8.07
MHM Estimate of PSS Case III	18.11	8.19
MHM Estimate of OL Case I	15.49	3.84
Unrestricted Intercepts, Unrestricted Linear Trends		
Monte Carlo Estimate	23.92	11.75
PSS Tables Case V	23.83	11.54
MHM Estimate of PSS Case V	23.94	11.64
MHM Estimate of OL Case2	18.40	3.84

TABLE FIVE: Rejection frequencies for $H_0 : r = 0$ against $H_1 : r \geq 1$ using different sets of critical values

	Trace Test	Maximum Eigenvalue Test
EViews case 3 specification and Osterwald-Lenum Case 1 critical values	12.01	6.88
EViews case 3 specification and PSS Case III critical values	5.59	5.25
EViews case 5 specification and Osterwald-Lenum Case 2 critical values	21.86	16.48
EViews case 5 specification and PSS Case V critical values	5.25	5.31

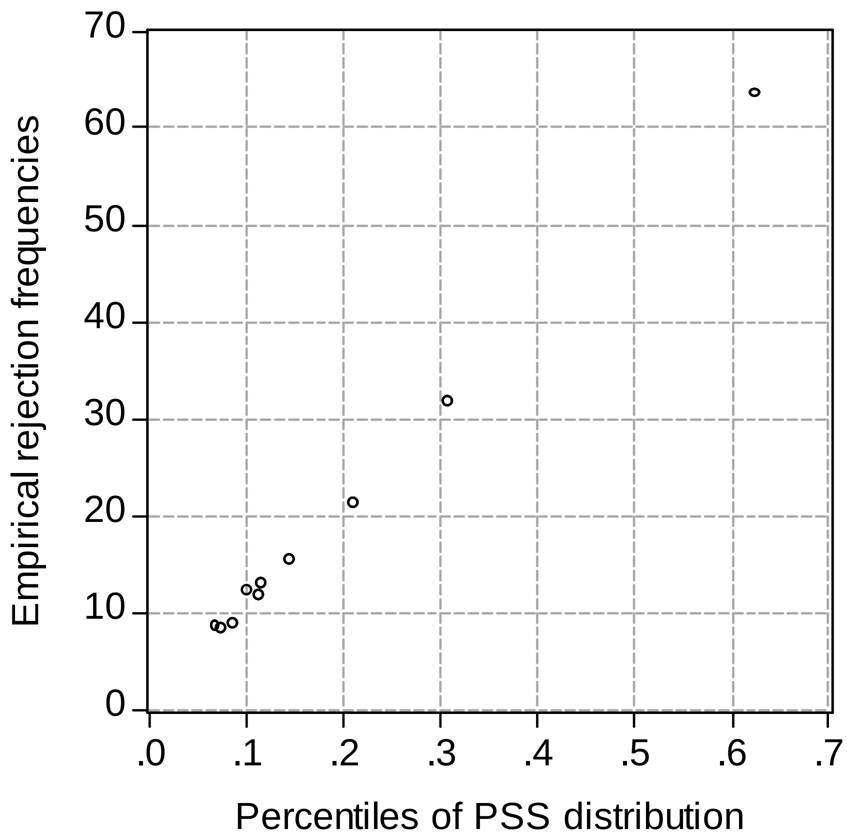
TABLE SIX: Rejection frequencies for $H_0 : r = \leq 1$ against $H_1 : r = 2$ using different sets of critical values

	Trace Test	Maximum Eigenvalue Test
EViews case 3 specification and Osterwald-Lenum Case 1 critical values	31.87	31.87
EViews case 3 specification and PSS Case III critical values	5.03	5.03
EViews case 5 specification and Osterwald-Lenum Case 2 critical values	63.67	63.67
EViews case 5 specification and PSS Case V critical values	5.04	5.04

TABLE SEVEN: Percentiles of the PSS distribution and empirical rejection frequencies

Number of variables in VECM	Osterwald-Lenum Case 1 Critical Value	Osterwald-Lenum Case 2 Critical Value	P-value for PSS case III distribution corresponding to Osterwald Lenum case 1 critical value	P-value for PSS case V distribution corresponding to Osterwald Lenum case 2 critical value	Rejection Frequency based on Osterwald-Lenum Case 1 Critical Value	Rejection Frequency based on Osterwald-Lenum Case 2 Critical Value
1	3.8415	3.8415	0.30534	0.62280	31.87	63.67
2	15.4947	18.3977	0.11169	0.20987	11.77	21.29
3	29.7971	35.0109	0.08361	0.14260	8.97	15.55
4	47.8561	55.2458	0.07221	0.11424	8.56	13.17
5	69.8189	79.3414	0.06654	0.09866	8.72	12.34

FIGURE ONE


70
60
50
40
30
20
10
0
Empirical rejection frequencies
.0
.1
.2
.3
.4
.5
.6
.7
Percentiles of PSS distribution